Student Learning Centre

# Probability

Flinders University inspiring achievement

**Probability** is a branch of mathematics that is widely used in the sciences, business and finance, and medicine. We use probability to determine the **chance** or **likelihood** of an event or events.

In probability an **event** is something that may or may not occur. If it is **impossible** that an event will occur, its probability is $0$. If an event is **certain** to occur, its probability is $1$. The probability of any other event is between these two values.

Probability can be calculated in two possible ways:

- **Experimental probability** by observing the results of experiments. Using this approach probability is calculated based on the number of times an event occurs written as a fraction of the number of repetitions. The disadvantage of experimental probability is that it is not possible to generalise because the results obtained will most likely differ from one experiment to the next. For this reason theoretical probability is used in probability calculations.

- **Theoretical probability** without actually carrying out the actions based on *ideal* outcomes.

The **sample space** of an event (or experiment) is the set of all possible **outcomes** that can occur. For example, if a coin is tossed, then the two possible outcomes are 'head' and 'tail'. The set of all possible outcomes is therefore $\{H, T\}$. This is called the sample space of the experiment and is denoted by $S$.

### Calculating theoretical probability

Let $E$ be an event and $n(E)$ be the number of ways that the event can occur. Let $S$ be the sample space and $n(S)$ be the total number of outcomes in the sample space. The probability of the event is written as $P(E)$.

$$\mathrm{p}(E) = \frac{number\ of\ elements\ in\ E}{number\ of\ elements\ in\ S} \qquad \text{or in symbols:} \qquad \mathrm{p}(E) = \frac{n(E)}{n(S)}$$

| Some Examples | | Explanation | Solution |
|---|---|---|---|
|  | Probability of getting either a 2 or 4 on a die. | Since there are two ways to get either a 2 or a 4, there are six possible outcomes. | $\mathrm{p}(2\ or\ 4) = 2/6\ or\ 1/3$ |
|  | A glass jar contains 6 red, 5 green, 8 blue and 3 yellow jellybeans. If a single jellybean is chosen at random from the jar, what is the probability of choosing a red? a green? a blue? a yellow jellybean? | $\mathrm{p}(red) = (no.of\ red)/(total\ no.of\ jellybeans)$<br>$\mathrm{p}(green) = (no.of\ green)/(total\ no.of\ jellybeans)$<br>$\mathrm{p}(blue) = (no.of\ blue)/(total\ no.of\ jellybeans)$<br>$\mathrm{p}(yellow) = (no.of\ yellow)/(total\ no.of\ jellybeans)$ | $\mathrm{p}(red) = 6/22 = 3/11$<br>$\mathrm{p}(green) = 5/22$<br>$\mathrm{p}(blue) = 8/22 = 4/11$<br>$\mathrm{p}(yellow) = 3/22$ |
|  | A spinner has six sectors with three different colours blue, red and yellow. What's the probability when it is spun it will land on blue? | Since there are two areas where blue can appear and there is six sectors in total then:<br>$\mathrm{p}(blue) = (no.of\ blue)/(total\ no.of\ sectors)$ | $\mathrm{p}(blue) = 2/6 = 1/3$ |

Table adapted from http://www.rapidtables.com/math/symbols/Basic_Math_Symbols.htm

## MUTUALLY EXCLUSIVE EVENTS

Let's look at calculating probabilities for more complex situations starting with **mutually exclusive** events. For mutually exclusive events it is **impossible** that they can happen at the same time. Very often you will read the word **'or'** to indicate that events are mutually exclusive.

### EXAMPLE 1

Think about rolling a pair of die and adding the sum of the numbers on the faces. The sum could be anything between $2$ and $12$, and there are $36$ possible ways the faces of the dice could be added. What is the probability that the sum will be a $7$ **OR** $11$? (These are mutually exclusive events because it is impossible for a sum of $7$ and $11$ to occur at the same time.)

- A sum of $7$ could be formed in the following ways: $(1,6)(2,5)(3,4)(4,3)(5,2)(6,1)$
  So $\mathrm{p}(sum\ of\ 7) = {}^{6}/_{36}$
- And a sum of $11$ could be formed in the following ways: $(5,6)(6,5)$
  So $\mathrm{p}(sum\ of\ 11) = {}^{2}/_{36}$

Therefore the probability of either a sum of 7 or a sum of $11 = {}^{6}/_{36} + {}^{2}/_{36} = {}^{8}/_{36} = {}^{2}/_{9}$

### EXAMPLE 2

In a pack of $52$ Cards what is the probability of getting a queen or an ace?
These are mutually exclusive events because it is impossible for a card to be a queen and an ace at the same time.

- There are $4$ queens in the pack so the probability of a Queen is ${}^{4}/_{52} = {}^{1}/_{13}$
- There are $4$ aces in the pack so the probability of an ace is ${}^{4}/_{52} = {}^{1}/_{13}$

Therefore the probability of a Queen **or** an Ace is written like this:

$$\mathrm{p}(Queen\ or\ Ace) = ({}^{1}/_{13}) + ({}^{1}/_{13}) = {}^{2}/_{13}$$

**For <u>mutually exclusive</u> events the probability of $A$ or $B$ is the sum of the probabilities:**

$$\mathbf{p}(\boldsymbol{A\ or\ B})\ \mathbf{p} = (\boldsymbol{A})\ \mathbf{p} + (\boldsymbol{B})$$

http://www.mathsisfun.com/data/probability-events-mutually-exclusive.html

What about when an event is non-mutually exclusive? Sometimes events are not mutually exclusive because they can occur at the same time.

### Example 3

What is the probability that when two die are rolled the sum will be less than $5$ or the faces will show the same number.

- Let event $A$ be that the number on each face is the same, therefore $A = (1,1), (2,5), (3,3), (4,4), (5,5), (6,6)$
- Let event $B$ be that the sum is less than 5, therefore $B = (1,1), (1,2), (1,3), (2,1), (2,2), (3,1)$

Event $A$ and $B$ can occur at the same time if the faces are $(1, 1)$, so it is important to count this outcome only once rather than once in $\mathrm{p}(A)$ and once in $\mathrm{p}(B)$. Therefore, the probability of a sum less than 5 or probability of both faces the same is $({}^{6}/_{36}) + ({}^{6}/_{36}) - ({}^{1}/_{36}) = ({}^{11}/_{36})$

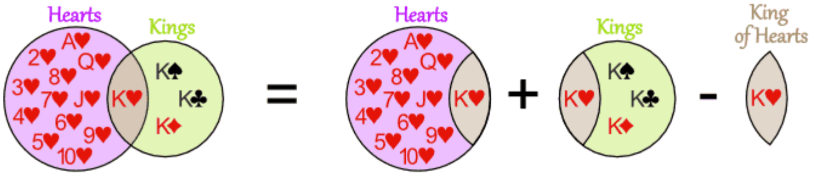
We need to subtract the probability of getting $(1, 1)$ so that we do not count this outcome twice.

**If events are not-mutually exclusive:**
$$\mathbf{p}(\boldsymbol{A\ or\ B}) = \mathbf{p}(\boldsymbol{A}) + \mathbf{p}(\boldsymbol{B}) - \mathbf{p}(\boldsymbol{A\ and\ B})$$

### Example 4

When picking from a pack of $52$ cards what is the probability of getting a king or a heart?

There are $13$ cards that are hearts and there are $4$ cards that are kings. But one card is both a king and a heart, so this event is not mutually exclusive.




$$\mathrm{p}(king\ or\ heart) = \left({}^{13}/_{52}\right) + ({}^{4}/_{52}) - ({}^{1}/_{52}) = {}^{16}/_{52} = {}^{4}/_{13}$$

## Compound Events

A compound event consists of two or more simple events. The events comprise a compound event that can be either independent events or dependent events.

- **Independent events** are events where the occurrence of one of the events **does not** affect the occurrence of the other event.
- **Dependent events** are events where the occurrence of one of the events **does** affect the occurrence of the other event.

To find the probability of two **independent events** occurring simultaneously or in sequence, find the probability of each event separately and then multiply these probabilities:

$$\mathrm{p}(A\ and\ B) = \mathrm{p}(A) \times \mathrm{p}(B)$$

## EXAMPLE 5

A coin is tossed and a single 6-sided die is rolled. Find the probability of landing on the head side of the coin and rolling a 3 on the die.

$\text{p(head)} = 1/2$

$\text{p(3 on die)} = 1/6$

$\text{p}(head\ and\ 3) = \text{p}(head) \times \text{p}(3) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$

## EXAMPLE 6

Two archers fire at a target simultaneously. Tiffany hits the target $^7/_{10}$ times and Saib hits the target $^2/_3$ times. What is the probability that they will both hit the target?

$\text{p}(Tiffany) = {^7/_{10}}$

$\text{p}(Saib) = {^2/_3}$

$\text{p}(Tiffany\ and\ Saib) = \frac{7}{10} \times \frac{2}{3} = \frac{14}{30} = \frac{7}{15}$

To find the probability of two **dependent events** find the probability of the first event, “adjust” the probability of the second event based on the fact that the first has occurred, and then multiply the probabilities.

## EXAMPLE 7

A card is chosen at random from a standard deck of 52 playing cards. Without replacing it, a second card is chosen. What is the probability that the first card chosen is a queen and the second card chosen is a jack?

$\text{p}(queen\ on\ first\ pick) = {^4/_{52}}$

To find $\text{p}(jack\ on\ 2^{nd}\ pick\ if\ queen\ picked\ on\ the\ 1^{st})$ remember that the first card (queen) was not put back in the pack, so now there are only 51 cards left.

$\text{p}(Jack\ on\ 2^{nd}\ pick\ if\ queen\ picked\ on\ the\ 1^{st}) = {^4/_{51}}$

$\text{p}(queen\ and\ jack) = \frac{4}{52} \times \frac{4}{51} = \frac{16}{2652} = \frac{4}{663}$

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